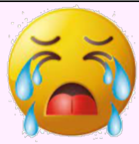
Hyperbolics

Hyperbolics				
Basics				
Definitions $\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\cosh x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$ $\text{sech}\, x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$ $\coth x = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$				
Useful Results $\sinh 0$ $\sinh(-x) = -\sinh x$ $\cosh 0 = 1$ $\cosh(-x) = \cosh x$				
Graphs				
Domain and Range				
Solving using exp definition				
Inverse				
$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right), x \in \mathbb{R}$				
Identities and Equations				
$\cosh^2 x - \sinh^2 x = 1$				
$\tanh x = \frac{\sinh x}{\cosh x}$				
$\tanh^2 x + \text{sech}^2 x = 1$				
$\coth^2 x - \text{csch}^2 x = 1$				
Differentiating				
$\sinh\ f(x) = \text{f}'(x)\cosh \text{f}(x)$ $\cosh\ f(x) \Rightarrow \text{f}'(x)\sinh \text{f}(x)$ $\tanh\ f(x) \Rightarrow \text{f}'(x)\text{sech}^2\, \text{f}(x)$ $\text{sech}\ f(x) \Rightarrow -\text{f}'(x) \tanh x\, \text{sech}\, x$ $\text{csch}\ f(x) \Rightarrow -\text{f}'(x) \coth x\, \text{csch}\, x$ $\coth\ f(x) \Rightarrow -\text{f}'(x) csch^2 x$ $\sinh^{-1} f(x) \Rightarrow \frac{f'(x)}{\sqrt{1+f(x)^2}}$ $\cosh^{-1} f(x) \Rightarrow \frac{f'(x)}{\sqrt{f(x)^2-1}}$ $\tanh^{-1} f(x) \Rightarrow \frac{f'(x)}{1-f(x)^2}$ $\text{sech}^{-1} f(x) \Rightarrow \frac{-f'(x)}{x\sqrt{1-(f(x))^2}}$ $\text{csch}^{-1} f(x) \Rightarrow \frac{-f'(x)}{\lvert x\rvert\sqrt{1+(f(x))^2}}$ $\coth^{-1} f(x) \Rightarrow \frac{f'(x)}{1-f(x)^2}$				
Integrating				
• $\int \frac{1}{\sqrt{a^2+(bx)^2}}\, dx = \frac{1}{b}\sinh^{-1}\left(\frac{bx}{a}\right) + c = \frac{1}{b}\ln\left(bx + \sqrt{(bx)^2 + a^2}\right) + c$ • $\int \frac{1}{\sqrt{(bx)^2-a^2}}\, dx = \frac{1}{b}\cosh^{-1}\left(\frac{bx}{a}\right) + c = \frac{1}{b}\ln\left(bx + \sqrt{(bx)^2 - a^2}\right) + c, x > a$ • $\int \frac{1}{a^2-(bx)^2}\, dx = \frac{1}{ab}\tanh^{-1}\left(\frac{bx}{a}\right) + c = \frac{1}{2ab} ln \left\lvert\frac{a+bx}{a-bx}\right\rvert + c, \lvert x\rvert < 1$ • $\int \frac{1}{(bx)^2-a^2}\, dx = \frac{1}{2ab} ln \left\lvert\frac{bx-a}{bx+a}\right\rvert + c$				
